Substitution and income effects for perfect complements

Consider a consumer who consumes two goods, X and Y, which are perfect complements. The utility function is given by:

$$U(X, Y) = \min(X, Y)$$

The initial prices of the goods are $P_X = 2$ and $P_Y = 1$. The consumer has an income of $M = 100$.

1. Find the initial consumption bundle: (X_0, Y_0)
2. Suppose the price of good X increases to $P'_X = 4$ while the price of good Y remains the same. Calculate the new consumption bundle (X_1, Y_1).
3. Calculate the substitution effect by finding the consumption bundle when the consumer is compensated to reach the same utility level as initially, with the new prices.
4. Calculate the income effect by finding the difference between the compensated bundle and the new consumption bundle.

Solution

1. The budget constraint is given by:
$$2X + Y = 100$$
Since the goods are perfect complements, the consumer will consume them in equal quantities, i.e., $X = Y$. Therefore, we can substitute $Y = X$ into the budget constraint:
$$2X + X = 100 \implies 3X = 100 \implies X_0 = \frac{100}{3} \approx 33.33$$
Thus, the initial consumption bundle is:
$$(X_0, Y_0) = \left(\frac{100}{3}, \frac{100}{3}\right)$$

2. With the new price of good X, the budget constraint becomes:
$$4X + Y = 100$$
Again, since $X = Y$:
$$4X + X = 100 \implies 5X = 100 \implies X_1 = 20$$
Thus, the new consumption bundle is:
$$(X_1, Y_1) = (20, 20)$$

3. To find the substitution effect, we need to determine the consumption bundle where the consumer can reach the same initial utility level $\left(\frac{100}{3}, \frac{100}{3}\right)$ with the new prices:
$$4X + Y = M'$$
$$5X = M'$$
$$X = M'/5$$
$$U(100/3, 100/3) = U(M'/5, M'/5)$$
$$100/3 = M'/5$$
$$166.666 = 500/3 = M'$$
Thus, the compensated consumption bundle is:
$$(X_1, Y_1) = (\frac{500}{3}\frac{1}{5}, \frac{500}{3}\frac{1}{5}) = (100/3, 100/3)$$
The substitution effect is the change from the initial bundle to the compensated bundle:
$$SE = 100/3 - 100/3 = 0$$

4. The income effect is the change from the compensated bundle to the new bundle:
$$IE = 20 - 100/3 = -13.333$$